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Non-Linear Analysis of the Free Electron Lasers Utilizing a Linear Wiggler Field

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A one-dimensional steady state FEL amplifier, utilizing a linearly polarized magnetic wiggler field, is analyzed in the linear and non-linear operating regimes. Our formulation of the FEL problem includes all the efficiency enhancements schemes, electron beam thermal effects and collective effects, i.e., space charge waves. The axial electron velocity, due to a linearly polarized wiggler field, has a spatially oscillatory component. It is shown that this axial velocity jiggle does not result in phase incoherence between the electrons and the ponderomotive wave. The axial velocity jiggle is explicitly incorporated

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into our analytical model. The inclusion of the electron jiggle velocity is shown to have a quantitative effect on the non-linear wave particle dynamics. The electron trapping potential associated with the ponderomotive wave is also derived. Finally, a number of numerical examples pertinent to the design of a 10 μm FEL amplifier are presented.

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CONTENTS

NON-LINEAR ANALYSIS OF THE FREE ELECTRON LASERS UTILIZING A LINEAR WIGGLER FIELD

I. INTRODUCTION

Free electron lasers (FELs) based on stimulated scattering from relativistic electron beams show great potential for becoming a new class of efficient devices capable of generating intense levels of coherent radiation. This class of FELs is characterized by a pump or wiggler field which is typically a spatially periodic magnetic field.

The magnetic wiggler field can be either linearly or circularly polarized. A circularly polarized wiggler is somewhat simpler to analyze because the axial particle velocity, for a fixed amplitude and period wiggler, is constant (independent of axial position). A linearly polarized wiggler, on the other hand, introduces a spatially oscillating term in the axial particle velocity. To our knowledge previous FEL analyses[1-32] have either taken the wiggler to be circularly polarized or have neglected the spatially oscillating part of the axial particle velocity with a linearly polarized wiggler.

Many of the future FEL experiments will employ a linearly polarized magnetic wiggler field. There are a number of practical advantages to this type of wiggler as opposed to a circularly polarized wiggler. These advantages are: i) relative simplicity of construction (which includes axial variation of the amplitude and period of the wiggler), ii) somewhat higher field amplitudes can be obtained and, iii) it is easier to obtain a linearly polarized radiation source for amplification.

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II. PHASE COHERENCE

In this paper we analyze the FEL in the steady state amplifying configuration. Our model for the FEL consists of a one dimensional relativistic electron beam propagating through a linearly polarized spatially periodic magnetic wiggler field $\underset{\sim}{B}_w(z)$, as shown in Fig. 1. The vector potential associated with the linearly polarized magnetic pump is

$$\underset{\sim}{A}_w(z) = A_w(z) \sin\int_0^z k_w(z')\, dz'\, \hat{e}_y \ , \tag{1}$$

where the amplitude $A_w(z)$ and period $\ell_w(z) = 2\pi/k_w(z)$ are slowly varying, known functions of z. The general temporal steady state radiation field and electrostatic (Coulomb) field excited by the interaction of the electron beam and wiggler field are respectively given by

$$\underset{\sim}{A}(z,t) = A(z) \sin\left(\int_0^z k(z')dz' - \omega t\right)\hat{e}_y \ , \tag{2a}$$

and

$$\phi(z,t) = \phi_1(z) \cos\left(\int_0^z \left(k(z') + k_w(z')\right) dz' - \omega t\right)$$

$$+ \phi_2(z) \sin\left(\int_0^z \left(k(z') + k_w(z')\right)dz' - \omega t\right) , \tag{2b}$$

where A, ϕ_1, ϕ_2, and k are assumed to be slowly varying functions of z compared to the radiation wavelength. Even for highly efficient FELs the radiation field is typically much less than the pump field, i.e., $|A| << |A_w|$. In this section we will consider some of the characteristics of a FEL with a linearly polarized magnetic pump field and take A_w, k_w and k to be independent of axial position.

In the presence of only the wiggler field the electron velocity is

$$v_x = 0 \,,$$

$$v_y = v_{o\perp} \sin k_w z \,, \qquad \text{(3a-c)}$$

$$v_z = v_{oz} + \frac{v_{o\perp}^2}{4v_o} \cos 2\, k_w z \,,$$

where $v_{o\perp} = |e|A_w/(\gamma_o m_o c)$ is the magnitude of the transverse wiggle velocity, $v_{oz} = v_o - v_{o\perp}^2/4v_o$ is the average axial velocity, v_o is the magnitude of the total velocity, i.e., $\gamma_o = (1 - v_o^2/c^2)^{-1/2}$. For the present it is sufficient to say that the frequency of the radiation field is such that phase coherence exists between the ponderomotive wave and the axial particle motion. That is, the axial phase velocity of the ponderomotive wave and the axial particle velocity must be matched so that an exchange of energy between the particles and radiation field can take place. The phase velocity of the ponderomotive wave is $v_{ph} = \omega/(k + k_w)$. By taking $k = \omega/c$ and equating v_{ph} to the average axial particle velocity v_{oz}, we find that the radiation frequency is given by

$$\omega = ck = (1 + \beta_o)\, \gamma_{zo}^2\, v_{oz} k_w \,, \qquad (4)$$

where $\gamma_{zo}^2 = \gamma_o^2/(1 + \beta_{o\perp}^2 \gamma_o^2/2)$ is the effective axial gamma factor, $\beta_{o\perp} = v_{o\perp}/c$. It will now be shown that the spatially oscillatory part of the axial particle velocity, the second term of (3c), does not lead to phase incoherence regardless of the magnitude or wavelength of the wiggler field. The condition for phase coherence is

$$(k + k_w)\, \delta z_{os} \ll \pi/2 \,, \qquad (5)$$

where $\delta z_{os} = \beta_{o\perp}^2/8k_w$ is the amplitude of the axial particle displacement associated with the spatially oscillatory part of the axial particle velocity. Using (4) we find that

$$(k + k_w)\ \delta z_{os} = \frac{1}{2}\left(1 - (1 + \beta_{o\perp}^2\gamma_o^2/2)^{-1}\right) < \frac{1}{2}\ , \qquad (6)$$

and is always less than 1/2 and typically $\approx 1/4$. Hence, the phase coherence condition in (5) is always satisfied even for arbitrarily strong pump fields. Therefore, the oscillatory part of the axial particle velocity in a linearly polarized pump can never result in phase incoherence.

III. LINEAR AND NON-LINEAR THEORY

a. Non-Linear Self-Consistent Formulation

In this section we formulate the 1-D non-linear theory of the FEL for a linearly polarized wiggler field. We include in our formulation of the problem: i) a spatially varying wiggler amplitude and period, ii) space charge fields, and iii) a D.C. accelerating field $E_{ac}(z) = -\partial\phi_{ac}/\partial z\,\hat{e}_z$. The present formulation is similar to our previous treatment of the circularly polarized wiggler problem.[26] We will show later that the applications of the D.C. accelerating field can modify the phase of the particles resulting in enhanced efficiency.[33-36] This method is equivalent to schemes in which the pump amplitude and/or period is varied as a function of z[26-28, 31].

The wave equations for A and ϕ are

$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) A_y(z,t) = -\frac{4\pi}{c} J_y(z,t) \quad , \tag{7a}$$

and

$$\frac{\partial^2 \phi(z,t)}{\partial z \partial t} = 4\pi\, J_z(z,t) \quad . \tag{7b}$$

The driving currents, J_y and J_z, are given by

$$\underset{\sim}{J}(z,t) = -|e| \int \frac{\underset{\sim}{p}}{\gamma(\underset{\sim}{p}) m_o} f(z,\underset{\sim}{p},t)\, d^3\underset{\sim}{p} \quad . \tag{8}$$

The thermal electron distribution function can be expressed in the following form

$$f(z,\underset{\sim}{p},t) = n_o \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{u_{zo} g_o(u_{zo})}{\gamma_o(u_{zo})} \; \delta(z - \tilde{z}(t_o, u_{zo}, t)) \tag{9}$$

$$\delta(p_y - \tilde{p}_y(t_o, u_{zo}, t))\,\delta(p_z - \tilde{p}_z(t_o, u_{zo}, t)) \; dt_o \; du_{zo} ,$$

where n_o is the uniform particle density for $z \leq 0$, which is to the left of the interaction region and hence outside of the wiggler field, $u_{zo} = p_{zo}/m_o$, p_{zo} is the axial electron momentum for $z \leq 0$, $g_o(u_{zo})$ is the distribution function associated with the initial spread in axial electron momentum, $\tilde{z}(t_o, u_{zo}, t)$ is the axial position at time t of the particle which crossed the $z = 0$ plane at time t_o with axial momentum p_{zo} and $\tilde{\underset{\sim}{p}}(t_o, u_{zo}, t)$ is the momentum vector at time t of the particle which crossed the $z = 0$ plane at time t_o with axial momentum p_{zo}. Substituting (9) into (8) and carrying out the integration over momentum, the driving current becomes

$$\underset{\sim}{J}(z,t) = -|e|n_o \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{u_{zo}}{\gamma_o(u_{zo})} \; g_o(u_{zo}) \; \frac{\tilde{\underset{\sim}{p}}(t_o, u_{zo}, t)}{\tilde{p}_z(t_o, u_{zo}, t)} \tag{10}$$

$$\delta(t - \tau(t_o, u_{zo}, z)) \; dt_o \; du_{zo} \quad ,$$

where

$$\tau(t_o, u_{zo}, z) = t_o + \int_o^z \frac{dz'}{\tilde{v}_z(t_o, u_{zo}, z')} \quad ,$$

is the time it takes a particle to travel to position z if it crossed the $z = 0$ plane at time t_o with momentum $p_{zo} = m_o u_{zo}$. Note that $\tilde{v}_z = \tilde{p}_z/\tilde{\gamma} m_o$ is the axial velocity of the particle with initial conditions t_o, u_{zo}.

Following the method presented in Ref. (26), we obtain the equations governing the amplitude and wavenumber of the radiation field and space charge field. Substituting (10) into (7), and multiplying the electromagnetic wave equation by $\begin{smallmatrix}\sin\\ \cos\end{smallmatrix}\left(\int_0^z (k(z')\,dz' - \omega t\right)$ and the electro-static wave equation by $\begin{smallmatrix}\sin\\ \cos\end{smallmatrix}\left(\int_0^z (k(z') + k_w(z'))\,dz' - \omega t\right)$ and integrating over one time period, we obtain the following equations for $A(z)$, $k(z)$, $\phi_1(z)$, and $\phi_2(z)$,

$$\left(\frac{\omega}{c} - k(z)\right) A(z) = -\frac{1}{2}\,\frac{\omega_b^2}{c^2}\,\frac{1}{\omega}\int_{-\infty}^{\infty}\frac{u_{zo}g_o(u_{zo})}{\gamma_o(u_{zo})}$$

$$\left[A_w(z)\left\langle\frac{\cos\tilde{\Psi}(t_o,u_{zo},z)}{\tilde{\gamma}(t_o,u_{zo},z)}\right\rangle + A(z)\right] du_{zo}\ , \tag{11a}$$

$$k^{1/2}(z)\,\frac{\partial}{\partial z}\left(A(z)k^{1/2}(z)\right) = \frac{1}{2}\,\frac{\omega_b^2}{c^2}\,\frac{1}{c}\int_{-\infty}^{\infty}\frac{u_{zo}g_o(u_{zo})}{\gamma_o(u_{zo})}$$

$$A_w(z)\left\langle\frac{\sin\tilde{\Psi}(t_o,u_{zo},z)}{\tilde{\gamma}(t_o,u_{zo},z)}\right\rangle du_{zo}\ , \tag{11b}$$

$$\phi_1(z) = -\frac{\omega_b^2}{c^2}\,\frac{c^2}{\omega^2}\,\frac{2m_oc}{|e|}\int_{-\infty}^{\infty}\frac{u_{zo}g_o(u_{zo})}{\gamma_o(u_{zo})}\left\langle\cos\tilde{\Psi}(t_o,u_{zo},z)\right\rangle du_{zo}, \tag{11c}$$

$$\phi_2(z) = -\frac{\omega_b^2}{c^2}\,\frac{c^2}{\omega^2}\,\frac{2m_oc}{|e|}\int_{-\infty}^{\infty}\frac{u_{zo}g_o(u_{zo})}{\gamma_o(u_{zo})}\left\langle\sin\tilde{\Psi}(t_o,u_{zo},z)\right\rangle du_{zo}, \tag{11d}$$

where $\omega_b = (4\pi|e|^2n_o/m_o)^{1/2}$,

$$\langle --- \rangle = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} dt_o(---) \text{ and } \tilde{\psi}(t_o, u_{zo}, z) = \int_0^z \left((k(z') + k_w(z') - \frac{\omega}{\tilde{v}_z}\right) dz' - \omega t_o$$

is the phase of the electron with respect to the beat wave (ponderomotive wave). If the electron beam can be considered to have negligible spread in axial momentum, upon entering the interaction region, we may replace $g_o(u_{oz})$ with a delta function, and carry out the integration over u_{oz}.

To complete the formulation of the FEL problem we require an equation describing the evolution of the phase $\tilde{\psi}(t_o, u_{zo}, z)$. To obtain this equation we first note that the y component of the particle's canonical momentum is a constant of motion. In terms of the Lagrangian independent variables z, t_o and u_{zo} the axial particle momentum is given by

$$\frac{d\tilde{p}_z}{dz} = \frac{-|e|^2}{2 m_o c^2 \tilde{v}_z \tilde{\gamma}^2}\left[\frac{\partial}{\partial z}(\underset{\sim}{A}_w(z) + \underset{\sim}{A}(z, t = \tau))^2\right.$$

$$\left. - 2\tilde{\gamma}\frac{m_o c^2}{|e|}\frac{\partial\phi(z, t = \tau)}{\partial z}\right] - \frac{|e|}{\tilde{v}_z} E_{ac}(z) , \qquad (12)$$

where $\tilde{p}_z = \tilde{p}_z(t_o, u_{oz}, t = \tau(t_o, u_{oz}, z)) = \tilde{\gamma}\,\tilde{v}_z m_o$. Equation (12) can be put into a more convenient and illuminating form by dropping terms which are not synchronous with the ponderomotive wave, using the well satisified inequality $|A| << |A_w|$ and assuming highly relativistic axial particle velocities. With these assumptions, together with the definition of the relative phase $\tilde{\psi}$, equation (12) takes the form of a generalized pendulum like equation

$$\frac{\partial^2\tilde{\psi}}{\partial z^2} = \frac{\partial k_w}{\partial z} - \frac{1}{4}\left(\frac{|e|}{\tilde{\gamma} m_o c^2}\right)^2 \frac{\omega}{c}\ \frac{\partial A_w^2}{\partial z} + \frac{|e|\ \omega/c}{m_o c^2\ \tilde{\gamma}\ \tilde{\gamma}_z^2}\ \frac{\partial \phi_{ac}}{\partial z}$$

$$+ \frac{\partial k}{\partial z} - \left(\frac{|e|}{\tilde{\gamma} m_o c^2}\right)^2 k\ k_w\left(A_w\ A \sin\tilde{\psi} + \frac{A_w^2}{2}\sin\left(2\int_o^z k_w\ dz'\right)\right)$$

$$+ \frac{2\omega_b^2/c^2}{\tilde{\gamma}\ \tilde{\gamma}_z^2}\left(<\cos\tilde{\psi}> \sin\tilde{\psi} - <\sin\tilde{\psi}> \cos\tilde{\psi}\right), \qquad (13)$$

where $\tilde{\gamma}_z = (1 - \tilde{v}_z^2/c^2)^{-\frac{1}{2}}$. In obtaining (13) we have assumed that all the particles have the same initial axial momentum, i.e., cold beam limit. The various terms affecting the phase in the generalized pendulum-like equation can now be distinguished. The first three terms represent the various efficiency enhancement methods available in the FEL. They include: i) tapering the wiggler wavelength, ii) tapering the wiggler amplitude and iii) D.C. accelerating potential. The equivalence of these schemes is evident. The fourth term is due to the self-consistent spatial variation of the radiation wavelength and can usually be neglected. The fifth and sixth terms represent: i) the ponderomotive wave due to the beating of the wiggler and radiation field and ii) the ponderomotive static wave due to the beating of the wiggler field with itself. If a circularly polarized wiggler field were chosen instead of a linearly polarized wiggler, the sixth term would not appear. The final term denotes the effects of space charge (collective) waves on the phase. The application of a D.C. accelerating potential can be an important method for enhancing the FEL's efficiency. If instead of a static magnetic wiggler field an electromagnetic wiggler is employed as a pump source, control of the wiggler amplitude or wavelength is not possible in a simple way. Application of a D.C. accelerating field is the most straightforward method for efficiency enhancement.

b. Trapping Potential.

One promising approach that can be taken to enhance efficiency is to initially trap a large fraction of the particles in the ponderomotive potential wells and adiabatically extract kinetic energy from the particles. To trap a substantial fraction of the electrons the trapping potential must be large or at least comparable to the initial spread in particle energy. This implies that a rather large amplitude radiation field must exist at the input to the interaction region. Using (14) it can be shown that the full trapping potential is given by $|e|\phi_{trap}/(\gamma_o m_o c^2) = (\frac{\Delta\gamma}{\gamma_o})_{trap} = 2\sqrt{2}\,\gamma_{zo}\beta_{o\perp}(A/A_w)^{1/2}$ where A is the radiation vector potential amplitude.

c. Linear Gain.

As a special case of (13) we consider the low gain, constant wiggler regime without space charge effects. Equation (13) reduces to the well known pendulum equation

$$\frac{d^2\tilde{\psi}}{dz^2} = -\left(\frac{|e|}{\gamma m_o c^2}\right)^2 k\, k_w \left(A_w A \sin\tilde{\psi} + \frac{A_w^2}{2}\sin(2\, k_w z)\right) , \tag{14}$$

where the radiation wavelength has also been taken as constant, i.e., $\partial k/\partial z = 0$. Using Eq. (11b) and (14) the small signal gain, i.e., $G = (A(z) - A(0))/A(0)$ for a linearly polarized wiggler field can be shown to be given by

$$G = -(1/16)\ \xi^2\beta_{o\perp}^2\ (k_w z)^3\ \partial(\sin\theta/\theta)^2/\partial\theta, \tag{15}$$

where $\theta = \mu z/2$, $\mu = (k + k_w) - \omega/v_{zo} = k_w(\omega_o - \omega)/\omega_o$, $\xi = \omega_b/\sqrt{\gamma_o}\,c\,k_w$. It is worth noting that the gain for a linearly polarized wiggler field is less than that for a circularly polarized wiggler by a factor of two. Except for this factor the gain expressions are identical. That is, the small signal gain for a linearly polarized wiggler[10] and for a circularly polarized wiggler[15,24-25,27] are identical if the amplitude of the latter is reduced by $\sqrt{2}$, i.e., if the amplitude of the circularly polarized wiggler is equal to the rms value of the amplitude of the linearly polarized wiggler.

The linear gain expression Eq. (15) does not take saturation effects into account. When the frequency mismatch is small,

$$0 < \frac{-\Delta\omega}{\omega_o} < \frac{\Delta\gamma}{\gamma_o}\bigg|_{trap} , \tag{16}$$

some particles are initially trapped, and the trapped particles cause saturation to occur within a short interaction length. Thus, the linear gain expression Eq. (15) should be used with discretion.

In what follows, the efficiency will be defined as the ratio of the electromagnetic energy flux increase to the initial electron kinetic energy flux

$$\eta(z) = \frac{1}{2}\left(\frac{|e|}{m_o c}\right)^2 \frac{\omega}{\omega_b^2}\left(\frac{k(z)A^2(z) - k(0)A^2(0)}{v_{zo}(\gamma_o - 1)}\right). \tag{17}$$

IV. ILLUSTRATIVE EXAMPLE OF A 10μm FEL

We now present a number of illustrative examples of an FEL utilizing a linearly polarized magnetic pump. In our examples we have chosen a 25 MeV electron beam ($\gamma_o = 50$), electron beam particle density of $n_o = 5 \times 10^{10} \mathrm{cm}^{-3}$, a magnetic wiggler field amplitude of $B_o = 5$ kG and wavelength of $\ell_w = 2.8$ cm. For these parameters, $A_w = 2.23 \times 10^3$ stat volts, $k_w = 2.24 \mathrm{cm}^{-1}$, $\beta o\perp =$ 2.61×10^{-2}, $\xi = 2.65 \times 10^{-2}$ and $\gamma_{zo} = 36.7$. The radiation wavelength is very nearly 10μm. As the input radiation source, we chose a high power CO_2 laser with a power flux of 5×10^7 W/cm^2, $A(0) = 0.1$ stat volts. Using the expression for the trapping potential we find that, with the above parameters, ~ 2% energy spread on the electron beam can be tolerated and substantial fraction of the electrons can still be trapped. In the following examples, Eqs. (11a-d) and (13) are solved numerically. Figure 2 shows the efficiency, for a constant parameter wiggler, as a function of axial position for $\Delta\omega/\omega_o = (\omega - \omega_o)/\omega_o = -1.25 \times 10^{-2}$ where $\omega_o = 2\gamma_z^2 c k_w$. The radiation field saturates at $z = 70\ \ell_w$ with a net gain of 0.4 and efficiency of 0.7%. Figure 3 shows the linear gain as a function of frequency mismatch, $\Delta\omega/\omega_o$. The dotted curve is the exact gain curve at $z = 30\ \ell_w$ in the linear regime of the interaction. The solid curve is the gain obtained from Eq. (15) evaluated at $z = 30\ \ell_w$.

In Fig. 4 the gain as a function of axial distance is shown for a frequency close to resonance, $\Delta\omega/\omega_o = -7 \times 10^{-3}$. The dotted curve is the exact gain obtained from the self-consistent non-linear equations, while the solid curve is the linear gain from Eq. (15). Since $\left.\frac{\Delta\gamma}{\gamma_o}\right|_{trap} = 2 \times 10^{-2}$, the inequality, Eq. (16), is satisfied, and particles are initially trapped. The trapped particles cause saturation to occur early in the interaction region and saturation effects result in a decrease in the gain as compared to the linear gain expression. The linear gain expression does not take trapping into account and therefore, the linear gain expression in (15) should be used with discretion.

The next two figures depict the gain and efficiency as a function of frequency mismatch for linearly polarized and circularly polarized wiggler fields. In making this comparison the amplitude of the circularly polarized wiggler is set equal to the rms amplitude of the linearly polarized wiggler, i.e., $B_{circular} = B_{linear}/\sqrt{2}$. Also, the amplitude of the circularly polarized radiation field is set equal to the rms amplitude of the linearly polarized radiation field. The particle equations of motion for the two sets of field polarization are identical except for the longitudinally oscillating term (sixth term in Eq. (13)) which is associated with the linearly polarized wiggler field. Figure 5 is a plot of gain at saturation versus frequency mismatch for both types of wiggler polarizations. Figure 6 shows the efficiency curve versus frequency mismatch again for both polarizations of fields. Figures 5 and 6 demonstrate that the longitudinal jiggle term induced in a linear wiggler can have a quantitative effect on the nonlinear FEL interaction.

As mentioned earlier, efficiency enhancement can be achieved using a number of schemes. The next figure shows an example in which the wiggler wavelength $\ell_w(z)$ and amplitude $B_w(z)$ are varied in such a way that the product $\ell_w B_w$ is held constant.[26-28,31] The frequency ω is chosen so that the ponderomotive wave is exactly resonant with the particles, i.e., $\Delta\omega = 0$. If none of the various efficiency enhancement schemes were employed, the gain would be zero. In Fig. 7, the tapering of the wiggler wavelength begins at the entrance of the interaction region. The period is changed from 2.8 cm at $z = 0$ to 2.66 cm at $z = 150\ \ell_w(0) = 420$ cm. The efficiency at the end of the interaction reached 1.15% as shown in Fig. 7. As seen from this figure the efficiency can be much larger by extending the interaction length. By increasing the power flux of the input CO_2 laser signal or wiggler amplitude, the trapping potential can be increased, permitting a more rapid rate of decrease of the wiggler period.

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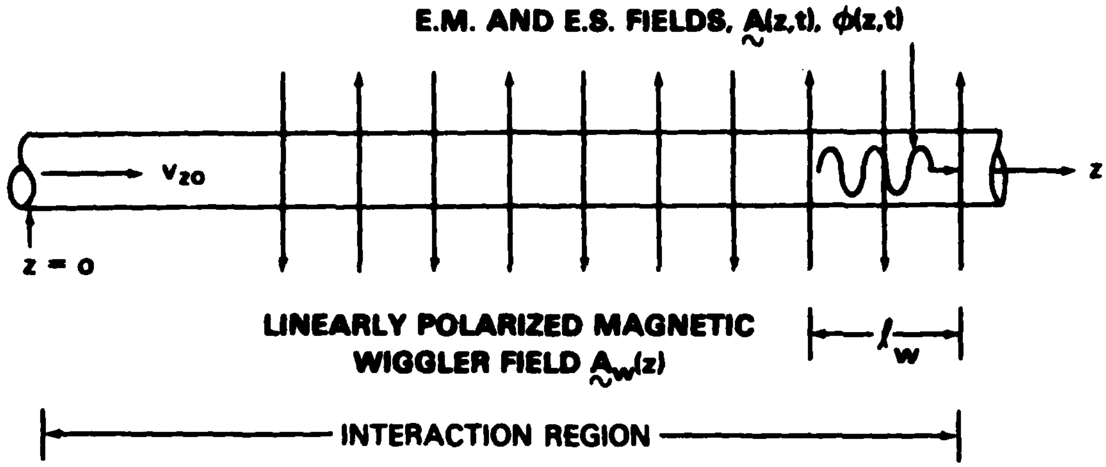

Fig. 1 - Schematic of the free-electron laser model. The unmodulated electron beam enters the interaction region from the left. The wiggler field builds up adiabatically and reaches a constant amplitude for $z > 0$.

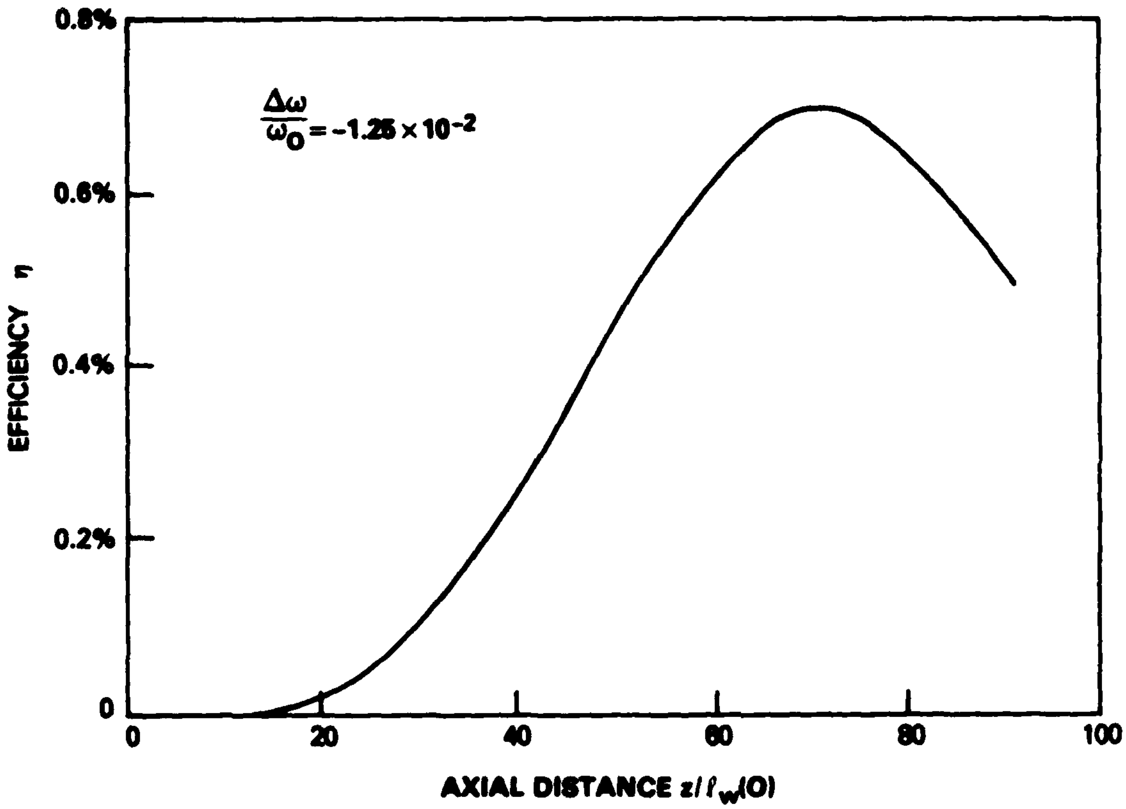

Fig. 2 - Efficiency versus normalized axial distance without efficiency enhancement for $\Delta\omega/\omega_o = -1.25 \times 10^{-2}$

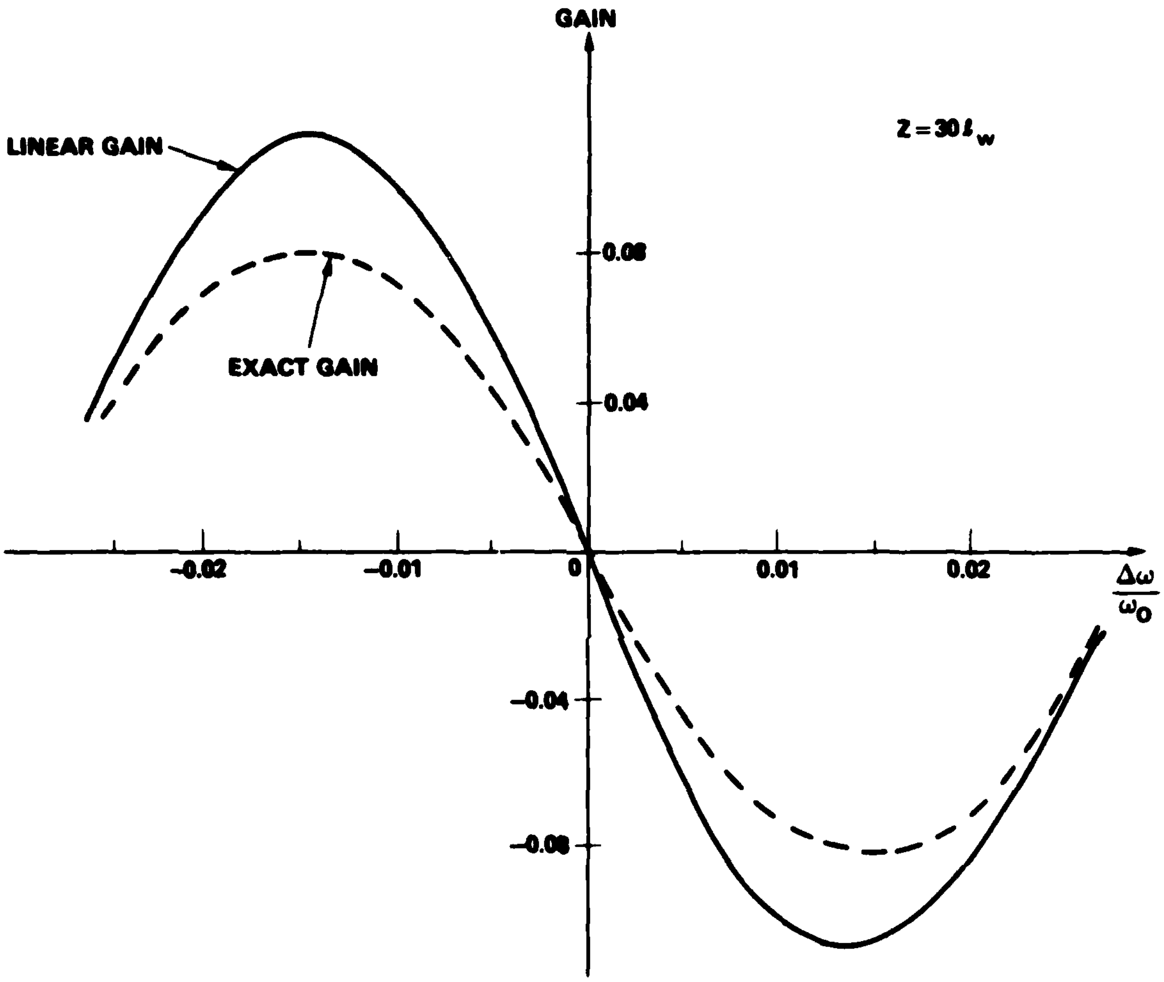

Fig. 3 - A comparison of the gain from linear gain expression (solid curve), and gain from non-linear calculation (dashed curve) as a function of frequency mismatch $\Delta\omega/\omega_o$ at $z = 30\ \ell_w(0)$

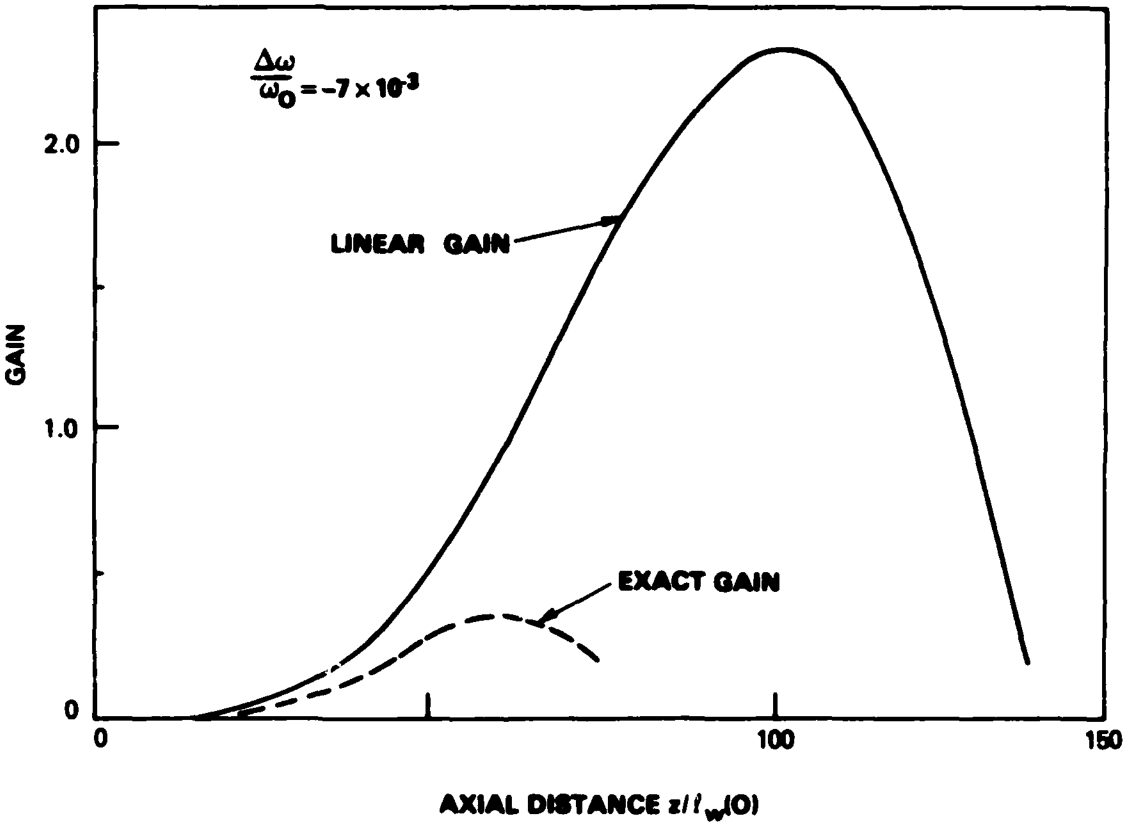

Fig. 4 - A comparison of the gain from linear gain expression (solid curve) and gain from non-linear calculation (dashed curve) as a function of axial distance for a small frequency mismatch $\Delta\omega/\omega_o = -7 \times 10^{-3}$

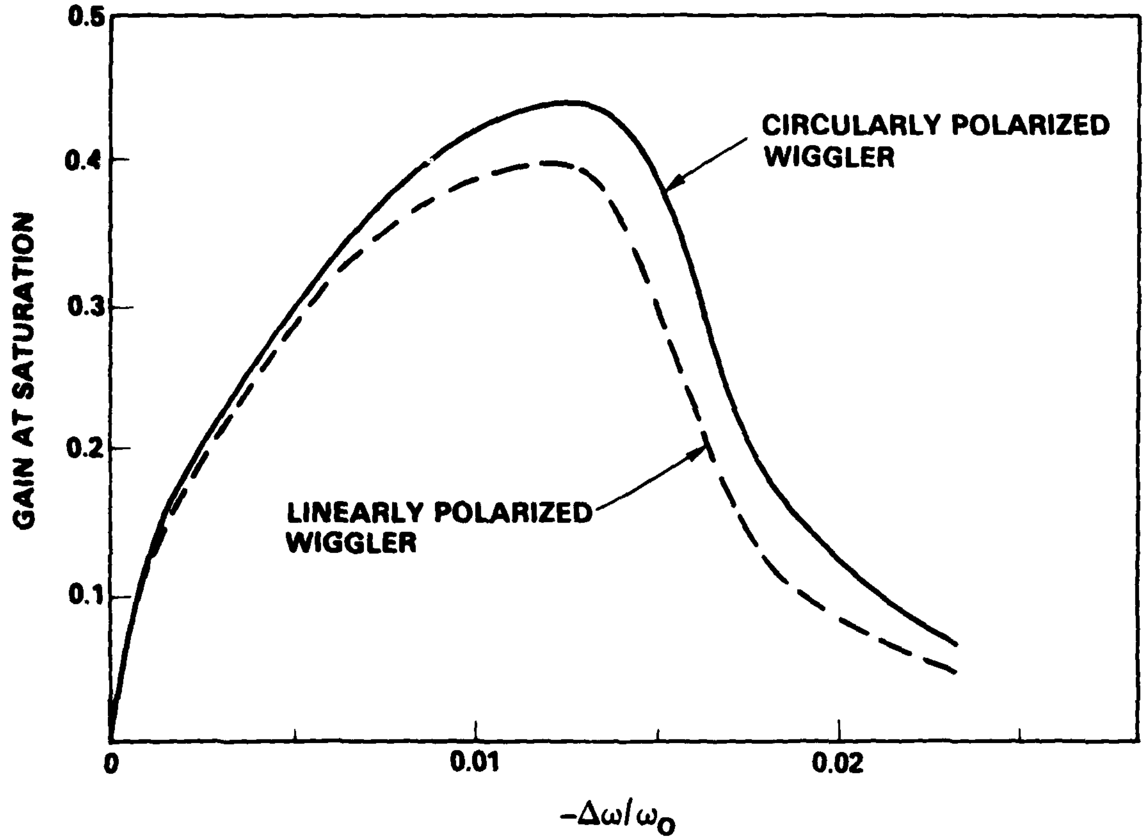

Fig. 5 - A comparison of gain at saturation using a linearly polarized wiggler (dashed curve) and circularly polarized wiggler (solid curve) versus frequency mismatch $-\Delta\omega/\omega_0$

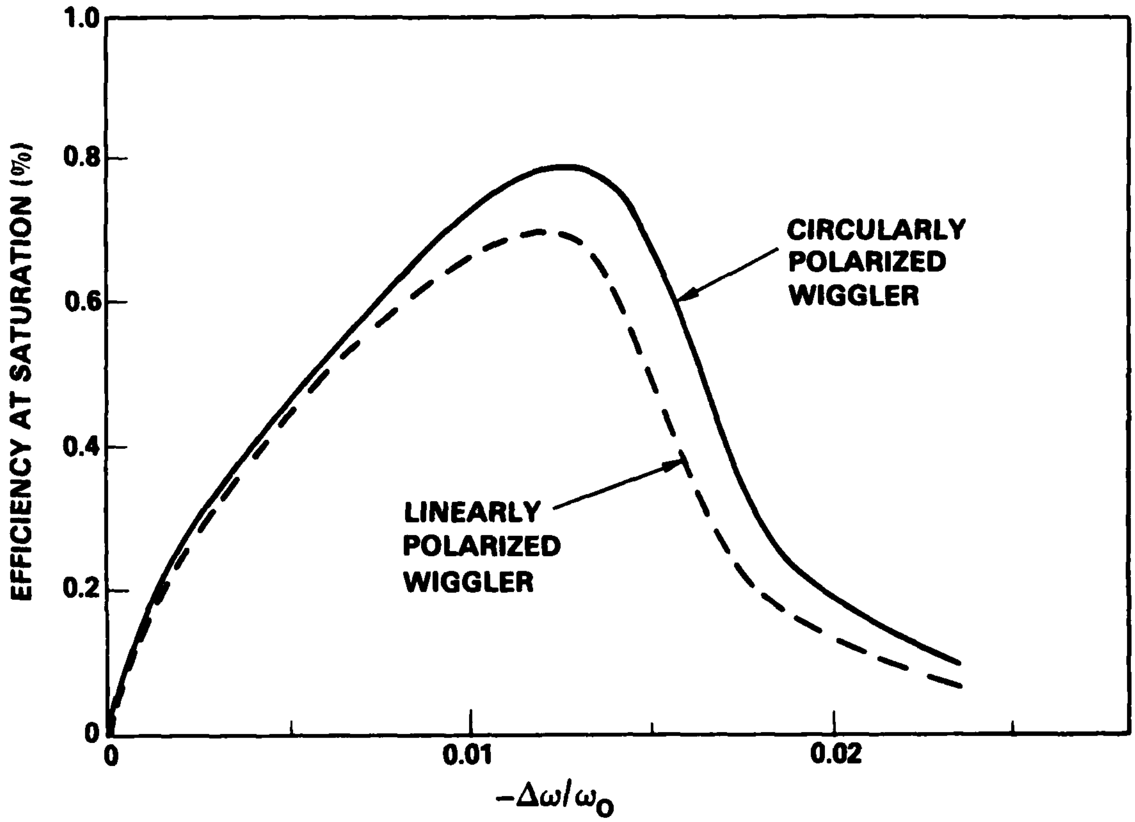

Fig. 6 - A comparison of efficiency at saturation using a linearly polarized wiggler (dashed curve) and circularly polarized wiggler (solid curve) versus frequency mismatch $-\Delta\omega/\omega_o$

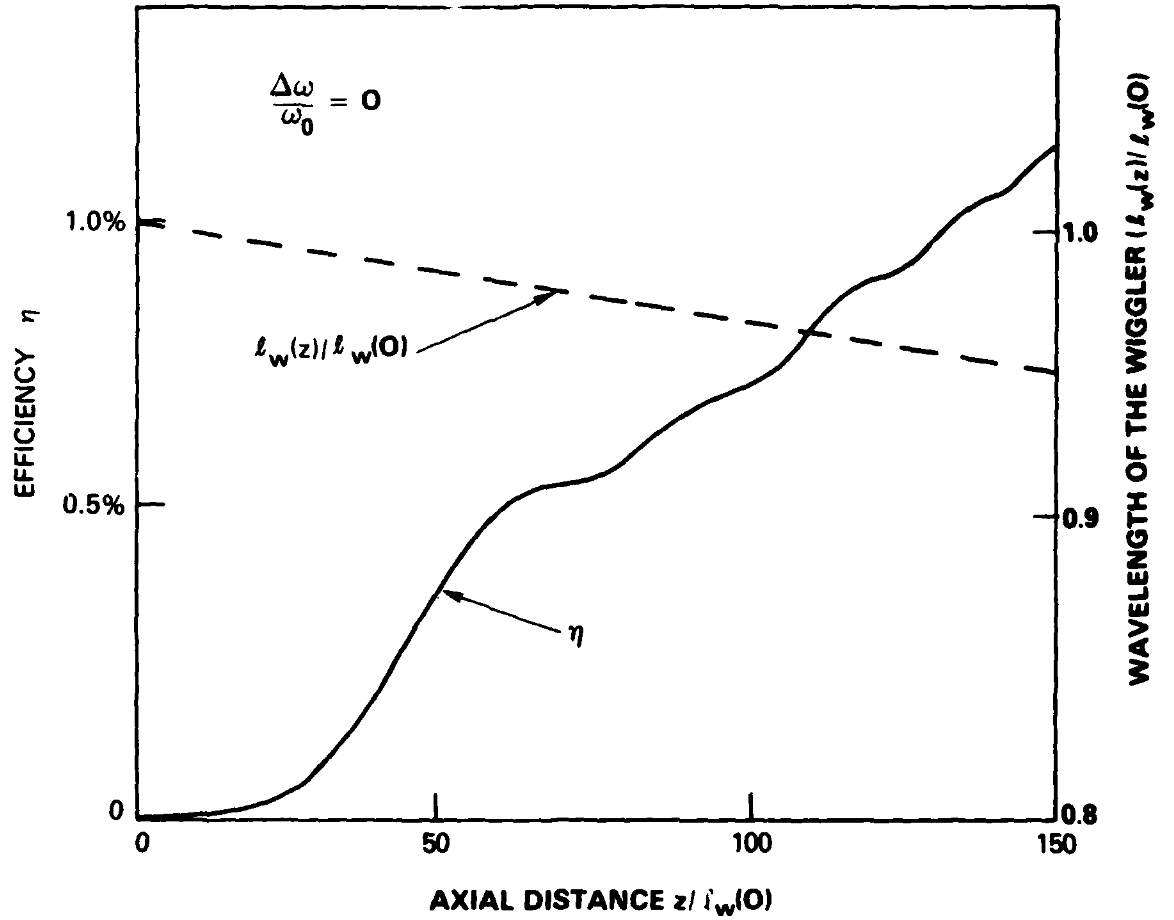

Fig. 7 - An example of efficiency enhancement by decreasing the magnetic wiggler period as shown for frequency that is on resonance. If the wiggler wavelength stayed constant, the saturation efficiency is approximately zero. The efficiency (solid curve) has increased to 1.15% at $z = 150\ \ell_w(0)$ with wiggler period $\ell_w(z)/\ell_w(0)$ (dashed curve) changing as shown.

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